Digital Logic Circuits

Lecture 5
Section 2.4

Robb T. Koether

Hampden-Sydney College

Wed, Jan 22, 2014

Outline

Babies and Alligators

(a) All babies are illogical.
(b) Nobody is despised who can manage a crocodile.
(c) Illogical persons are dispised.

- What conclusion can we draw from these premises?

Mermaids and Voyages

Mermaids and Voyages

(a) None of the unnoticed things, met with at sea, are mermaids.

(b) Things entered in the log, as met with at sea, are sure to be worth remembering.

(c) I have never met with anything worth remembering, when on a voyage.

(d) Things met with at sea, that are noticed, are sure to be recorded in the log.

- What conclusion can we draw from these premises?

Mermaids and Voyages

Mermaids and Voyages

(a) No interesting poems are unpopular among people of real taste.
(b) No modern poetry is free from affectation.
(c) All your poems are on the subject of soap-bubbles.
(d) No affected poetry is popular among people of real taste.
(e) No ancient poem is on the subject of soap-bubbles.

- What conclusion can we draw from these premises?

Outline

Logic Gates

- There are three basic gates.
  - AND-gate
  - OR-gate
  - NOT-gate
- Two other gates.
  - NAND-gate
  - NOR-gate

AND-Gate

p	q	Output
1	1	1
1	0	0
0	1	0
0	0	0

- An AND-gate.
  - The output is 1 if both inputs are 1.
  - The output is 0 if either input is 0.

OR-Gate

p	q	Output
1	1	1
1	0	1
0	1	1
0	0	0

- An OR-gate.
  - The output is 1 if either input is 1.
  - The output is 0 if both inputs are 0.

NOT-Gate

p	Output
1	0
0	1

- A NOT-gate.
  - The output is 1 if the input is 0.
  - The output is 0 if the input is 1.

NAND-Gate

p	q	Output
1	1	0
1	0	1
0	1	1
0	0	1

- An NAND-gate.
  - The output is 0 if both inputs are 1.
  - The output is 1 if either input is 0.

NOR-Gate

p	q	Output
1	1	0
1	0	0
0	1	0
0	0	1

- An NOR-gate.
  - The output is 0 if either input is 1.
  - The output is 1 if both inputs are 0.

Outline

Disjunctive Normal Form

- A logical expression is in disjunctive normal form (DNF) if
  - It is a *disjunction* of clauses,
  - Each clause if a *conjunction* of variables and negations of variables.
  - Each variable or its negation appears in each clause exactly once.

Example

$$p \rightarrow q \equiv (p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q).$$
$$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q).$$
$$p \mid q \equiv (p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q).$$
$$p \downarrow q \equiv \sim p \wedge \sim q.$$

- What are disjunctive normal forms for T and F?

Conjunctive Normal Form

- A logical expression is in conjunctive normal form (CNF) if
  - It is a *conjunction* of clauses,
  - Each clause if a *disjunction* of variables and negations of variables.
  - Each variable or its negation appears in each clause exactly once.

Example

$$p \rightarrow q \equiv \sim p \vee q.$$
$$p \leftrightarrow q \equiv (p \vee \sim q) \wedge (\sim p \vee q).$$
$$p \mid q \equiv \sim p \vee \sim q.$$
$$p \downarrow q \equiv (p \vee \sim q) \vee (\sim p \vee q) \vee (\sim p \vee \sim q).$$

- What are conjunctive normal forms for T and F?

Outline

Output Tables

Input		Output
1	1	0
1	0	1
0	1	0
0	0	0

- An output table shows the output of a logical function for every possible combination of inputs.

Designing Circuits

- To design a circuit that represents a logical function,
  - Write an output table for the circuit. The table reveals the DNF form of the function.
  - Write the logical expression and simplify it, if possible.
  - Draw the circuit using AND-gates, OR-gates, and NOT-gates (and NAND-gates and NOR-gates).

Example

Input		Output
1	1	0
1	0	1
0	1	0
0	0	0

- Design a circuit for the above function (which is $\sim (p \to q)$).

Example

- Design a circuit for $(p \rightarrow q) \wedge (q \leftrightarrow \sim r)$.

Example

Input			Output
p	q	r	
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	0
0	1	0	1
0	0	1	1
0	0	0	0

- Produce the output table for $(p \to q) \wedge (q \leftrightarrow \sim r)$.

Example

- Based on the output table, the DNF of

$$(p \rightarrow q) \wedge (q \leftrightarrow \sim r)$$

  is

$$(p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r).$$

- I do not see any way to simplify this.
- Draw the circuit.

Example

- Design a logic circuit for
$$(p \wedge q) \vee (\sim q \wedge \sim r) \vee r.$$
- Use the *conjunctive* normal form of
$$(p \wedge q) \vee (\sim q \wedge \sim r) \vee r$$
to design a circuit.

Outline

Assignment

Assignment

- Read Section 2.4, pages 64 - 75.
- Exercises 1, 2, 5, 6, 9, 10, 15, 17, 18, 19, 24, 25, 32, page 76.